**Analysis for one and two bystanders with continuous**

Hi! The main result of the previous section is that the bystander effect will be more present in situations where few people are selfish than in situations where most people are. To reach this conclusion the analysis assumes that there are only two types of bystanders: those who are selfish and those who are not. This assumption makes the model more tractable and helps us isolate the effect of changing the share of selfish bystanders, ceteris paribus. In reality, however, this assumption seems quite unreasonable. There are probably not only two types of bystanders, but a continuum of types. There may be some bystanders whose preferences for helping are really high, some whose preferences are average, and some who almost do not care about helping.

This section shows that the main intuition gained from the previous results holds even when we allow preferences for helping to be continuous. For simplicity, we assume that bystanders’ preferences for helping stem only from pure altruism: bystanders only care about whether the victim is helped, and not about whether they help themselves. We perform the analysis assuming that the cost of helping is the same across all bystanders. Bystanders’ types are defined by their pure altruism preference for seeing that the victim receives help, which is drawn from a commonly known continuous distribution function. We assume the density function of the distribution function to be S-shaped, such as the normal distribution.

Note that in this setup the share of selfish bystanders—those bystanders who do not help even when they are alone—is the share of bystanders whose pure altruism preference of seeing that the victim is helped is lower than the cost of helping. Thus, to change the share of selfish bystanders one must change either the pure altruism distribution function or the cost of helping. For simplicity, we will study the bystander effect between a group of one and two bystanders by changing the cost of helping. Note that, given a distribution function of pure altruism preferences, a low cost of helping implies that there is a low share of selfish bystanders, and a high cost of helping implies that there is a high share of selfish bystanders.

In line with our previous results, we show that there exists a unique cost —that corresponds to a determined share of selfish bystanders—that makes the bystander effect equal to zero. This means that given this cost the victim is equally likely to be helped in a group of one than in a group of two bystanders. We show that any cost that is lower than —where the share of selfish bystanders is thus lower—implies a positive bystander effect, meaning that victims are more likely to be helped in a group of one than in a group of two bystanders. On the other hand, any cost that is higher than —where the share of selfish bystanders is thus higher—implies a negative bystander effect, meaning that victims are more likely to be helped in a group of two than in a group of one bystanders.

**Model**

Bystanders ’s utility function is now if he helps, if another bystander helps, and if neither him nor another bystander helps. We assume that that is drawn from a commonly known probability distribution function with a cumulative distribution function bounded in the interval . We assume that is an S-shaped density function which is strictly convex for and strictly concave for , where . We further assume that .

This model focuses on whether a victim is more likely to be helped in a group of one or in a group of two bystanders. We therefore define the bystander effect as , where and are respectively the probabilities that the victim is helped when there are one and two bystanders.

**Theorem 3.** There exists a unique such that . Furthermore, for and for .

*Proof.* Note that if bystander is the only bystander who can help the victim, he will help whenever his type is higher than . This implies that, a priori, the probability that the victim is helped when only one bystander can help her is the probability that . This means that .

We look for a Bayesian Nash equilibrium to find . Denote by and the two bystanders. In this case, the condition that bystanders help as long as is no longer satisfied. To see this, denote by the probability that bystander helps (before the type realization). Because bystanders help as long as the utility of helping is higher than the utility of not helping, bystander helps if , and does not help if . Note that cannot be a Nash Equilibrium, since because and it may be that for some player . Then, if such player knows that , he will have incentives to deviate and help with sure probability. We can thus conclude that , which confirms that .

This means that the probability that at least one bystander helps is the probability that at least one picks a such that , or .

Putting and together,

Note that is uniquely defined by and . To find it, recall that bystander helps if and only if

Because is the probability that bystander picks a ,

Note that bystander will be indifferent between helping and not helping only if . Therefore, is found by equating both sides

which simplifies to

In what follows, we will derive the result by using , and the properties of .

Equation implies that when . Since , the condition holds for which becomes by . Defining for , it follows that

We will now derive the properties of and prove the result by using . Recall that is an S-shaped density function which is strictly convex for and strictly concave for , where . Since and , and because is S-shaped, there exists a unique point such that for , which we will denote . Then, has the following properties:

1. **is continuous in**  since both and are continuous.
2. **for ; for and ; and for** , by definition of and , and since is S-shaped.
3. **Let . Such is exists and is unique.**

*Proof.* We first note that , which are the values of for which is concave. To see that, note that any maximum of must satisfy the condition that . Because , this implies that in such maximum

To show that , suppose that holds for . Because is strictly convex in this range, by definition of convexity, for any and for ,

Dividing both sides by and reordering yields

that, by taking the limit as , becomes

Now pick and . Then, becomes and, as ,

which contradicts .

This implies that, if exists, it does when . To show that exists, notice that , , and for . Since is continuous, it follows that there exists at least one value of such that .

To show that is unique, note that

Because by every maximum satisfies the condition , introducing this condition into yields

which is negative since , and for . These conditions follow from strict concavity of for and the fact that .

Since is negative when the FOC is satisfied, this means that any critical point of in is a maximum, implying that such maximum is unique.

1. **for and for** . It follows from the previous item that for and for . To show that also for , notice that , and that this expression is positive by .

Now we have all the properties that we needed from to prove that there exists a unique such that , for and for .

1. **There exists a such that** .

*Proof.*Recall that . Since is continuous for , and , is also continuous. Hence, we need to show that, given , there exists a cost such that is negative and a cost such that is positive.

First, pick a such that . Note that since , there exists only one satisfying the condition . Since , and by the properties derived for , , for , and for . Since , it follows that . This implies that .

Second, pick a such that that corresponds to a such that is satisfied. Because we have previously shown that has a unique maximum, and because , . This implies that .

1. **is unique and for and for** .

*Proof.* By for , is injective in this interval. Note however that is non-injective in the interval since , , for and continuity. In particular, because has a unique critical point, for every there exists a unique such that and . This implies that any such that can only exist when (notice that since and ). To see that, note that such that for because is injective in this interval. Furthermore, such that for because in this interval and . Thus, .

To show that is unique, we will show that for any . The proof showing for any is analogous.

Define such that it satisfies . Pick a such that with a corresponding . Since for , and belongs to this interval, . Since , and , then . There are three possible cases:

1. If , then from for , . This implies that .
2. If , then because takes its maximum value at , and since is unique, for any . Thus .
3. If , since and for , . Thus , which completes the proof.